UNIT III – THREE PHASE CIRCUITS AND NETWORK TOPOLOGY

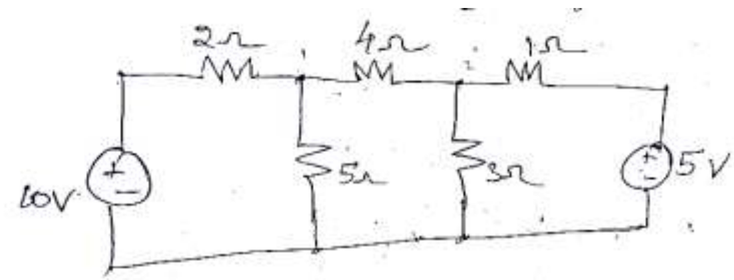
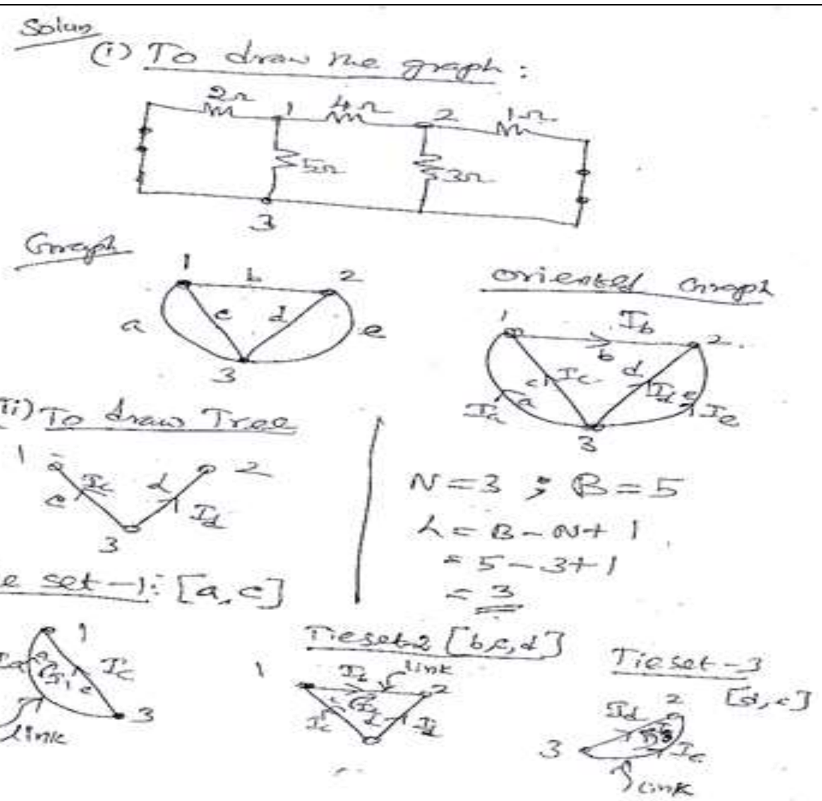
PART – A (2 Mark Questions With Key)

S.No	Questions	Mark	COs	BTL
1	**Give the relationship between the line and phase values in star connection.**		3	K1
	The relation between line voltage and phase voltage in a star connection is $\mathbf{V_L = \sqrt{3}V_{ph}}$ The relation between line current and phase current in a star connection is $\mathbf{I_L = I_{ph}}$	2		
2	**Give the relationship between the line and phase values in delta connection.**			
	The relation between line voltage and phase voltage in a delta connection is $\mathbf{V_L = V_{ph}}$ The relation between line current and phase current in a star connection is $\mathbf{I_L = \sqrt{3}I_{ph}}$	2	3	K2
3	**List the advantages of three phase system over single phase system.**			
	(i) Generation, transmission and distribution of 3 Phase power is more economical (ii) Three phase machines have better power factor and efficiency (iii) Three phase motors are self-starting (iv) For same size, the capacity of 3 phase machine is high	2	3	K1
4	**Define balanced supply and unbalanced supply.**			
	The three phase supply is said to be balanced when all three phase voltages are equal in magnitude and separated by 120˚ each other. Otherwise the system is said to be an unbalanced one.	1 1	3	K1
5	**Define balanced load and unbalanced load.**			
	The three phase load is said to be a balanced load when all the three load impedances are identical and hence the load current in all three phases are equal in magnitude and separated by 120˚ each other. Otherwise the load is said to be an unbalanced one.	1 1	3	K1
6	**What is meant by network topology?**			
	Network topology is the study of the properties of the network which are unaffected when we stretch, twist or distort the size and shape of the network.	2	3	K1
7	**Define graph and oriented graph.**			
	Graph of a network consists of nodes and branches of the network. In network the branches have elements but in the graph the branches are drawn by lines. When arrows are placed on the branches of the graph, it is called oriented graph.	1 1	3	K1
8	**Define planar network and non-planar network.**			
	A network is said to be planar, if it can be drawn on a plane surface without any crossovers. A network is said to be non-planar, if it cannot be drawn on a plane surface without any crossovers.	1 1	3	K1
9	**What is sub-graph and directed graph?**			
	When some of the branches in an original graph are removed the resultant graph is called sub-graph If every branch of a graph has a direction, then the graph is called as a directed graph.	1 1	3	K1
10	**Define duality or dual networks.**			
	If two electrical networks are governed by same type of equations then they are known as dual networks. Duality is a concept of forming (or identifying) a voltage basis network for a given current basis network (or vice versa) with similar form of governing equations and solutions.	2	3	K1
11	**What is a tree and co-tree?**			
	A tree is a connected sub-graph of a network, which consists of all the nodes of the original graph, but no closed paths. The complementary set of branches of the tree is called as the co-tree of the graph.	1 1	3	K1
12	**List the properties of the tree.**			
	(i) Tree contains all the nodes of the graph (ii) Tree contains (N-1) branches (i.e. Twigs) (iii) Tree does not have a closed path (iv) Tree is a sub-graph and complement of a co-tree.	2	3	K1
13	**What is meant by links, chords and twigs?**			
	The branches removed to form a tree are called links or chords. Branches of co-tree are known as links.	1	3	K1

<table>
<tr><td></td><td>The branches of a tree are called twigs.</td><td>1</td><td></td><td></td></tr>
<tr><td>14</td><td>Define complete incidence matrix and reduced incidence matrix.</td><td></td><td></td><td></td></tr>
<tr><td></td><td>The incidence matrix with information of all the nodes is called complete incidence matrix.
In an incidence matrix one of the row can be deleted or eliminated and such a matrix known as reduced incidence matrix.</td><td>1

1</td><td>3</td><td>K1</td></tr>
<tr><td>15</td><td>Distinguish between tie set and cut set.</td><td></td><td></td><td></td></tr>
<tr><td></td><td><table><tr><th>TIE SET</th><th>CUT SET</th></tr><tr><td>Tie set is a set of branches that form a closed path in a graph such that the closed path contains only one link(or chord) & reminder are tree branches (twigs)</td><td>Cut set contains one twig and the remaining branches are links</td></tr><tr><td>no. of tie sets= no. of links (L)
L=B-N+1</td><td>no. of cut sets= no. of twigs= N-1</td></tr></table></td><td>2</td><td>3</td><td>K1</td></tr>
<tr><td colspan="5">PART – B (12 Mark Questions with Key)</td></tr>
<tr><th>S.No</th><th>Questions</th><th>Mark</th><th>COs</th><th>BTL</th></tr>
<tr><td>1</td><td>Determine the line current, power factor, total power, reactive power and apparent power when a 3 phase 400 V supply is given to a balanced star connected load of impedance (15+j20) ohm in each branch.</td><td>12</td><td rowspan="2">3</td><td rowspan="2">K3</td></tr>
<tr><td></td><td>$Z + I_{ph} + I_L + pf$
$P + Q + S$

✓ $Z_{ph} = \sqrt{(R^2+X_L^2)}$= 25 ohms
✓ V_L = 400V $V_{ph} = V_L/\sqrt{3}$ = 230.95 V
✓ $I_{ph} = I_L = V_{ph} / Z_{ph}$ = 9.24A
✓ Pf = R/Z = 0.6 (lag)
✓ $P = \sqrt{3}\, V_L\, I_L \cos\theta$ = 3840.88 watts
✓ $Q = \sqrt{3}\, V_L\, I_L \sin\theta$ = 5121.18 VAR
✓ $S = \sqrt{3}\, V_L\, I_L$ = 6401.47 VA</td><td>2+1+1+2
2+2+2</td></tr>
<tr><td>2</td><td>i) Derive the relationship between line and phase voltages in a star connected system.
ii) A balanced star connected load of 3+j4 ohms in each phase is connected to a 3 phase 400V supply. Find phase and line currents. Also find total power consumed by the load.</td><td>7
5</td><td rowspan="2">3</td><td rowspan="2">K3</td></tr>
<tr><td></td><td>i) Circuit + phasor diagram + Derivation
ii) $Z + I_{ph} + I_L + pf + P$
$Z_{ph} = \sqrt{(R^2+X_L^2)}$= 5 ohms
V_L = 400V $V_{ph} = V_L/\sqrt{3}$ = 230.95 V
$I_{ph} = I_L = V_{ph} / Z_{ph}$ = 46.19 A
Pf = R/Z = 0.6 (lag)
$P = \sqrt{3}\, V_L\, I_L \cos\theta$ = 19200 watts</td><td>2+2+3
5*1=5</td></tr>
<tr><td>3</td><td>i) Derive the relationship between line and phase voltages in a delta connected system.
ii) A balanced delta connected load of 8-j6 ohms in each phase is connected to a 3 phase 230V supply. Find phase and line currents. Also find total power consumed by the load.</td><td>7
5</td><td rowspan="2">3</td><td rowspan="2">K3</td></tr>
<tr><td></td><td>i) Circuit + phasor diagram + Derivation
ii) $Z + I_{ph} + I_L + pf + P$
$Z_{ph} = \sqrt{(R^2+X_c^2)}$= 10 ohms
$V_L = 230V = V_{ph}$
$I_{ph} = V_{ph} / Z_{ph}$ = 23 A
$I_L = \sqrt{3}\, I_{ph}$ = 39.84 A
Pf = R/Z = 0.8 (lead)
$P = \sqrt{3}\, V_L\, I_L \cos\theta$ = 12696.53 watts</td><td>2+2+3
5*1=5</td></tr>
<tr><td>4</td><td>Determine the line current, power factor, total power, reactive power and apparent power when a 3 phase 400 V 50 Hz supply is given to a balanced delta connected load</td><td>12</td><td>3</td><td>K3</td></tr>
</table>

	consisting of 16 ohms resistor in series with 38.2 mH inductor in each branch.			
	$Z + I_{ph} + I_L + pf$ $P + Q + S$ ✓ $X_L = 2\pi f L = 12$ ohms ✓ $Z_{ph} = \sqrt{(R^2+X_c^2)} = 20$ ohms ✓ $V_L = 400\ V = V_{ph}$ ✓ $I_{ph} = V_{ph} / Z_{ph} = 20\ A$ ✓ $I_L = \sqrt{3}\ I_{ph} = 34.64\ A$ ✓ Pf = R/Z = 0.8 (lag) ✓ $P = \sqrt{3}\ V_L\ I_L \cos\theta = 19200$ watts ✓ $Q = \sqrt{3}\ V_L\ I_L \sin\theta = 14400$ VAR ✓ $S = \sqrt{3}\ V_L\ I_L = 24000$ VA	2+1+1+2 2+2+2		
5	**Determine the branch currents and voltages of the network given using Tie-set schedule.** 	12	3	K3
		4		

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Tie set schedule.

Loop Currents	Branches a	b	c	d	e
I_1	+1	0	-1	0	0
I_2	0	+1	+1	-1	0
I_3	0	0	0	-1	+1

$$M = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$I_B = \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \\ I_e \end{bmatrix} \quad I_L = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \quad V_B = \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \end{bmatrix}$$

$$I_B = M^T I_L \quad \text{———— (1)}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \\ I_e \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$= \begin{bmatrix} I_1 \\ I_2 \\ -I_1 + I_2 \\ -I_2 - I_3 \\ I_3 \end{bmatrix}$$

$\therefore I_a = I_1$, $I_b = I_2$, $I_c = -I_1 + I_2$, $I_d = -I_2 - I_3$, $I_e = I_3$ ———— (3)

By applying KVL to tiesets

$$M V_B = 0 \quad \text{———— (2)}$$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \end{bmatrix} = 0$$

$$\begin{bmatrix} V_a - V_c \\ V_b + V_c - V_d \\ -V_d + V_e \end{bmatrix} = 0$$

$$\left.\begin{aligned} V_a - V_c &= 0 \\ V_b + V_c - V_d &= 0 \\ -V_d + V_e &= 0 \end{aligned}\right\} \text{———— (4)}$$

4

Only branch a & branch e are having the voltage source.

By using Ohm's law:

$V_a = ?\quad V_b = ?\quad V_c = ?\quad V_d = ?\quad V_e ?$

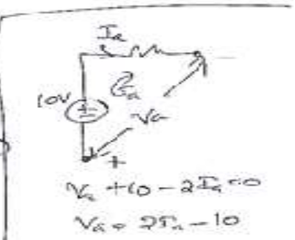
$V_b = I_b \times 4 = 4I_b$

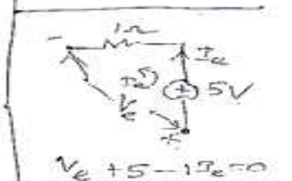
$V_c = I_c \times 5 = 5I_c$

$V_d = I_d \times 3 = 3I_d$

$V_a = 2I_a - 10$ — (5)

$V_e = I_e - 5$ — (6)





Using (5) & (6) &
Sub in (4).

(4) $\Rightarrow 2I_a - 10 - 5I_c = 0$

$4I_b + 5I_c - 3I_d = 0$

$-3I_d + I_e - 5 = 0$

$$\left.\begin{aligned} 2I_a - 5I_c &= 10 \\ 4I_b + 5I_c - 3I_d &= 0 \\ -3I_d + I_e &= 5 \end{aligned}\right\} \text{— (7)}$$

Using eqn (3)

$2I_1 - 5(-I_1 + I_2) = 10$

$4I_2 + 5(-I_1 + I_2) - 3(-I_2 - I_3) = 0$

$-3(-I_2 - I_3) + I_3 = 5$

$2I_1 + 5I_1 - 5I_2 = 10$

$4I_2 - 5I_1 + 5I_2 + 3I_2 + 3I_3 = 0$

$3I_2 + 3I_3 + I_3 = +5$

$7I_1 - 5I_2 = 10$

$-5I_1 + 12I_2 + 3I_3 = 0$

$3I_2 + 4I_3 = 5$

In matrix form

$$\begin{bmatrix} 7 & -5 & 0 \\ -5 & 12 & 3 \\ 0 & 3 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 5 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 7 & -5 & 0 \\ -5 & 12 & 3 \\ 0 & 3 & 4 \end{vmatrix} = 7(48-9) + 5(-20-0) + 0 = 7 \times 39 - 100 = 173$$

$$\Delta_1 = \begin{vmatrix} 10 & -5 & 0 \\ 0 & 12 & 3 \\ 5 & 3 & 4 \end{vmatrix} = 10(48-9) + 5(0-15) + 0 = 10(39) - 75 = 315$$

$$\Delta_2 = \begin{vmatrix} 7 & 10 & 0 \\ -5 & 0 & 3 \\ 0 & 5 & 4 \end{vmatrix} = 7(0-15) - 10(-20-0) + 0 = -105 + 200 = \underline{95}$$

$$\Delta_3 = \begin{vmatrix} 7 & -5 & 10 \\ -5 & 12 & 0 \\ 0 & 3 & 5 \end{vmatrix} = 7(12\times5-0) + 5(-25-30) + 0 = 420 - 275 = \underline{165}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{315}{173} = \underline{1.821\ A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{95}{173} = \underline{0.5491\ A}$$

$$\therefore \frac{\Delta_3}{\Delta} = \frac{145}{173} = \underline{0.838\ A}$$

loop currents are.

$$\boxed{\begin{aligned} I_1 &= 1.821\ A \\ I_2 &= 0.5491\ A \\ I_3 &= 0.8382\ A \end{aligned}}$$

Branch currents are.

$$\boxed{\begin{aligned} I_a &= I_1 = 1.821\ A \\ I_b &= I_2 = 0.5491\ A \\ I_c &= -I_1 + I_2 = -1.821 + 0.5491 = -1.2720 \\ I_d &= -I_2 - I_3 = -0.5491 - 0.8382 = -1.3873\ A. \\ I_e &= I_3 = 0.8382\ A \end{aligned}}$$

4

Branch voltages.

$$V_a = 2I_a - 10 = 2\times1.821 - 10 = \underline{-6.3584}\ V$$

$$V_b = 4I_b = 4\times0.5491 = 2.1964\ V$$

$$V_c = 5I_c = 5\times(-1.2717) = -6.3585\ V$$

$$V_d = 3I_d = 3\times(-1.3873) = -4.1619\ V$$

$$V_e = I_e - 5 = 0.8382 - 5 = -4.1618\ V$$

[The negative sign indicates the actual direction of currents & polarity of voltages are just opposite to the assumed direction]

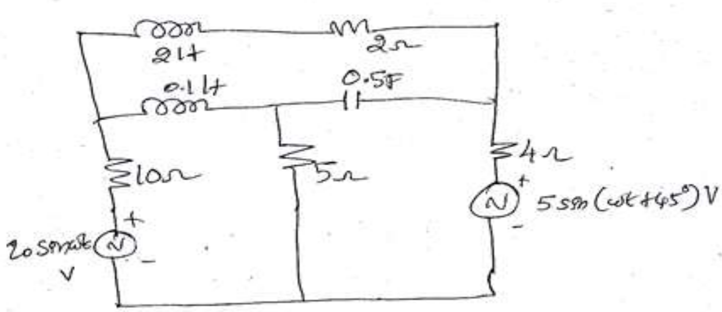
Result:

(i) Branch currents - - -

(ii) Branch voltages . - - -

<table>
<tr><td>6</td><td>Find the line currents for the unbalanced delta connected load of impedances Z_{RY}=20+j30Ω, Z_{YB}=6-j10Ω, Z_{BR}=12+j10Ω are connected to a 3 phase 200V supply, if the phase sequence is (i) RYB and (iii) RBY</td><td>12</td><td rowspan="2">3</td><td rowspan="2">K3</td></tr>
<tr><td></td><td>
(i) RYB PHASE SEQUENCE
$$I_{RY} = \frac{200\angle 0}{20+j30} = 5.55\angle -56.31A = 3.08 - j4.62A$$
$$I_{YB} = \frac{200\angle -120}{6-j10} = 17.15\angle -60.96A = 8.32 - j15A$$
$$I_{BR} = \frac{200\angle 120}{12+j10} = 12.8\angle 80.2A = 2.18 + j12.6A$$
I_R= I_{RY}-I_{BR}= 0.9-j17.22= 17.24 A
I_Y= I_{YB}-I_{RY}= 5.24-j10.38= 11.63 A
I_B= I_{BR}-I_{YB}= -6.14+j27.6= 28.27 A

(ii)RBY PHASE SEQUENCE
$$I_{RY} = \frac{200\angle 0}{20+j30} = 5.55\angle -56.31A = 3.08 - j4.62A$$
$$I_{YB} = \frac{200\angle 120}{6-j10} = 17.15\angle 179.04A = -17.14 + j0.28A$$
$$I_{BR} = \frac{200\angle -120}{12+j10} = 12.8\angle -159.8A = -12 - j4.42A$$
I_R= I_{RY}-I_{BR}= 15.08-j0.2= 15.08 A
I_Y= I_{YB}-I_{RY}= -20.22+j4.9= 20.8 A
I_B= I_{BR}-I_{YB}= 5.14-j4.7= 6.96 A
</td><td>4</td></tr>
</table>

<table>
<tr><th colspan="5">PART – C (20 Mark Questions with Key)</th></tr>
<tr><th>S.No</th><th>Questions</th><th>Mark</th><th>COs</th><th>BTL</th></tr>
<tr><td>1</td><td>Determine the DUAL of the Network and also verify it.
</td><td>20</td><td>4</td><td>K4</td></tr>
</table>

Solun

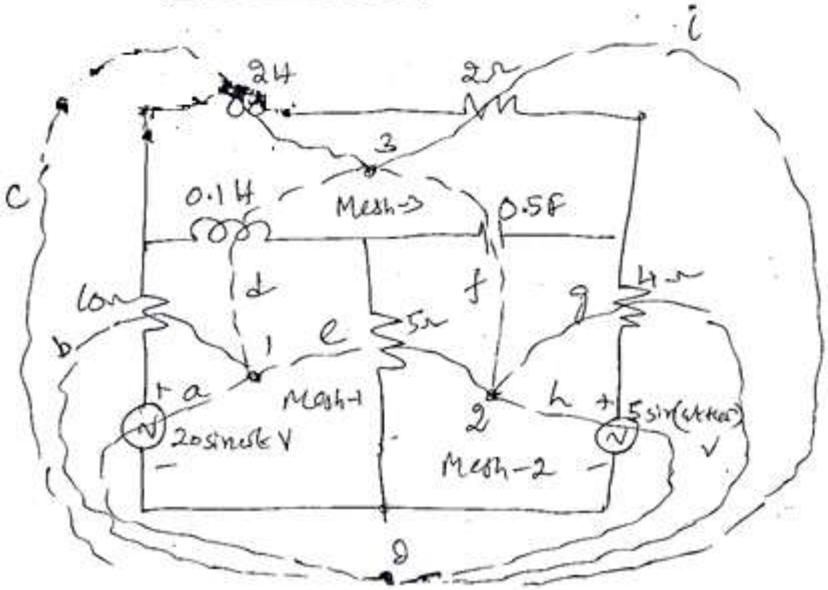
3 meshes 1, 2 & 3.

Mesh currents i_1, i_2 & i_3.

Draw dual N/w & then form the eqns.

To draw a Dual N/w :

ORIGINAL N/w



Branch	Elements of the branch in original N/w	Elements of branch in dual N/w	connection of branch in dual N/w
a	20 sin ωt V voltage source	20 sin ωt A current source	Node 1 → Node 0
b	10Ω resistance	10 ℧ conductance	1 → 0
c	2H Inductance	2F Capacitance	3 → 0
d	0.1H Inductance	0.1F Capacitance	1 → 3
e	5Ω Resistance	5 ℧ conductance	1 → 2
f	0.5F Capacitance	0.5H Inductance	2 → 3
g	4Ω resistance	4 ℧ conductance	2 → 0
h	5 sin(ωt+45) V voltage source	5 sin (ωt+45) A current source	2 → 0

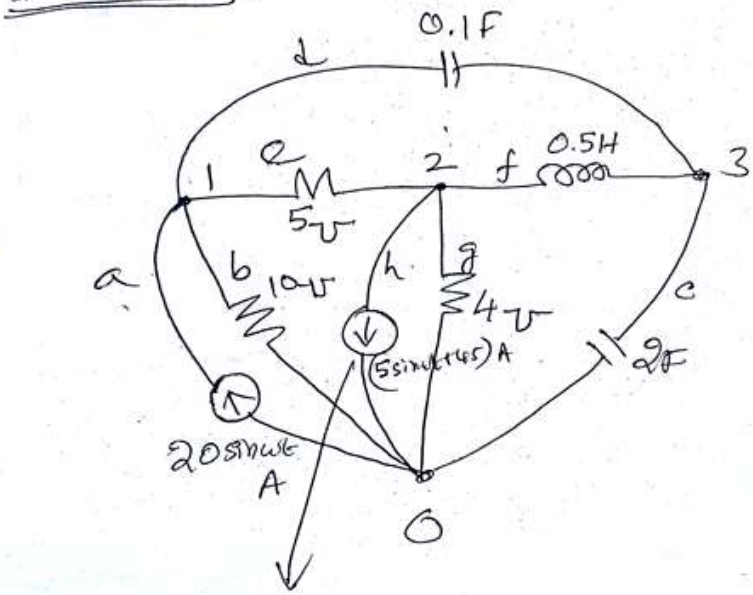
Original N/w with meshes and nodes(4)

Dual N/w(4)

Table(4)

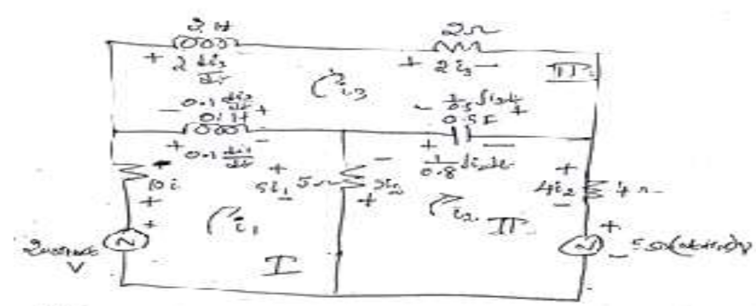
Verification:
Mesh eqns(4)
Node eqns(4)

Dual N/ω :



Original N/ω is a Mesh Basis N/ω
So its dual will be a Node basis N/ω

To find Mesh Equations :



Mesh Equations are

Mesh – I

$$20\sin\omega t = 10 i_1 + 0.1\frac{di_1}{dt} - 0.1\frac{di_3}{dt} + 5i_1 - 5i_2$$

i.e.,

$$20\sin\omega t = 10 i_1 + 0.1\frac{d}{dt}(i_1 - i_3) + 5(i_1 - i_2) \quad \text{--- (1)}$$

Mesh II

$$-5\sin(\omega t+45) = 5i_2 - 5i_1 + \frac{1}{0.5}\int i_2\,dt - \frac{1}{0.5}\int i_3\,dt + 4i_2$$

i.e.,

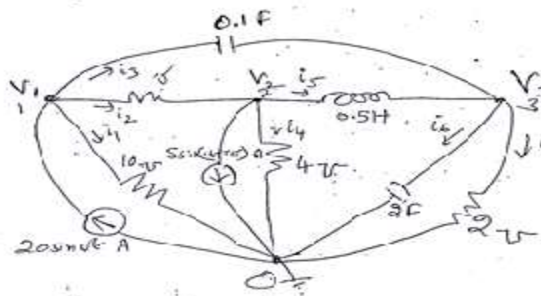
$$-5\sin(\omega t+45) = 5(i_2 - i_1) + \frac{1}{0.5}\int (i_2 - i_3)\,dt + 4i_2 \quad \text{--- (2)}$$

$$0 = 2\frac{di_3}{dt} + 2i_3 + \frac{1}{0.5}\int i_3 dt - \frac{1}{0.5}\int i_2 dt + 0.1\frac{di_3}{dt} - 0.1\frac{di_1}{dt} \quad -2$$

i.e.,

$$0 = 2\frac{di_3}{dt} + 2 i_3 + \frac{1}{0.5}\int (i_3 - i_2)dt + 0.1\frac{d}{dt}(i_3 - i_1) \quad (3)$$

Node equations of the Dual N/w:



Apply KCL at all Nodes:

$$i_c = C\frac{dv}{dt}$$

Node-1:

$$20\sin\omega t = i_1 + i_2 + i_3$$

$$= 10(V_1 - 0) + 5(V_1 - V_2) + 0.1\frac{d}{dt}(V_1 - V_3)$$

$$20\sin\omega t = 10V_1 + 5(V_1 - V_2) + 0.1\frac{d}{dt}(V_1 - V_3) \quad (4)$$

Node-2:

$$i_2 = 5\sin(\omega t + 45) + i_4 + i_5$$

$$-5\sin(\omega t + 45) = -i_2 + i_4 + i_5$$

$$= -5(V_1 - V_2) + 4(V_2 - 0) + \frac{1}{0.5}\int (V_2 - V_3)dt$$

$$-5\sin(\omega t + 45) = 5(V_2 - V_1) + 4V_2 + \frac{1}{0.5}\int (V_2 - V_3)dt \quad (5)$$

Node-3:

$$i_3 + i_5 = i_6 + i_7$$

$$\therefore 0 = i_6 + i_7 - i_3 - i_5$$

$$0 = 2\frac{d(V_3 - 0)}{dt} + 2(V_3 - 0) - 0.1\frac{d(V_1 - V_3)}{dt} - \frac{1}{0.5}\int (V_2 - V_3)dt$$

$$0 = 2\frac{dV_3}{dt} + 2V_3 + \frac{1}{0.5}\int (V_3 - V_2)dt + 0.1\frac{d}{dt}(V_3 - V_1) \quad (6)$$

Compare eqns (1), (2), (3) with eqns (4), (5) & (6).

It is observed that the mesh equations of the original network & Node equations of the dual network are in identical form.

2	**A balanced three phase load has an impedance of 7+j7 ohms in each phase. The load is fed with 3 phase 415V supply. The phase sequence is RYB. Determine the line current, phase current, power factor, power, reactive volt-ampere and volt-ampere if the load is (i) star connected (ii) delta connected.**	20	3	K3
	Star: I_L + pf + P + Q + S	5*2=10		
	Delta: I_L + pf + P + Q + S	5*2=10		
	Star connection ✓ $Z_{ph} = \sqrt{(R^2+X_L^2)}$= 9.9 ohms ✓ V_L = 415V $V_{ph} = V_L/\sqrt{3}$ = 239.6 V ✓ $I_{ph} = I_L = V_{ph} / Z_{ph}$ = 24.2 A ✓ Pf = R/Z = 0.707 (lag) ✓ $P = \sqrt{3}\ V_L\ I_L \cos\theta$ = 12298 watts ✓ $Q = \sqrt{3}\ V_L\ I_L \sin\theta$ = 12298 VAR ✓ $S = \sqrt{3}\ V_L\ I_L$ = 17394.5 VA Delta connection ✓ $Z_{ph} = \sqrt{(R^2+X_L^2)}$= 9.9 ohms ✓ V_L = 415 V = V_{ph} ✓ $I_{ph} = V_{ph} / Z_{ph}$ = 41.92 A ✓ $I_L = \sqrt{3}\ I_{ph}$ = 72.6 A ✓ Pf = R/Z = 0.707 (lag) ✓ $P = \sqrt{3}\ V_L\ I_L \cos\theta$ = 36893.7 watts ✓ $Q = \sqrt{3}\ V_L\ I_L \sin\theta$ = 36893.7 VAR ✓ $S = \sqrt{3}\ V_L\ I_L$ = 52183.43 VA			