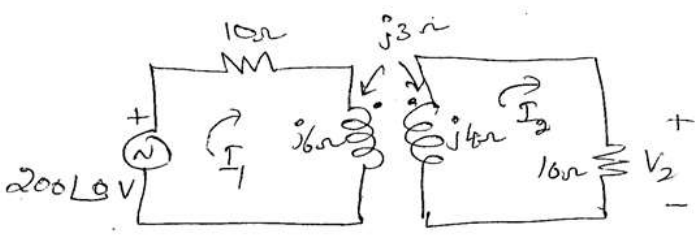
UNITV– RESONANCE AND COUPLED CIRCUITS

PART – A (2 Mark Questions With Key)

S.No	Questions	Mark	COs	BTL
1	**When is a circuit said to be in resonance?**		3	K2
	A circuit is said to be in resonance, when the current in the circuit is in phase with the applied voltage.	2		
2	**What is frequency response?**	2	3	K2
	The response of a circuit with sinusoidal excitation as a function of the angular frequency, ω is known as the frequency response.			
3	**What is resonance frequency?**	2	3	K2
	The frequency at which resonance occurs is called resonant frequency. $f_r = \frac{1}{2\pi\sqrt{LC}}$			
4	**What is bandwidth?**	2	3	K2
	Bandwidth of the system is the range of frequencies for which, the current or output voltage is equal to 70.7% of its value at the resonant frequency and it is denoted by BW. BW= f_2-f_1			
5	**What is selectivity of a resonant circuit?**	2	3	K2
	Selectivity is the response of the resonant circuit to certain frequencies and eliminate all other frequencies. If the bandwidth is narrow, the selectivity will be very high. Selectivity= $Bandwidth / resonant\ frequency = \frac{1}{Q}$			
6	**What is quality factor?**	2	3	K2
	The quality factor (Q) is the ratio of the resonant frequency to bandwidth. $Q = 2\pi \frac{Maximum\ energy\ stored}{Energy\ dissipated\ per\ cycle}$ $Q = \frac{L\omega_r}{R} = \frac{1}{C\omega_r R} = \frac{1}{R}\sqrt{\frac{L}{C}}$			
7	**When are two circuits said to be coupled?**	2	3	K2
	Two circuits are said to be coupled when energy transfer takes place from one circuit to the other when one of the circuit is energized.			
8	**Define self-inductance.**	2	3	K2
	Flux linkage in a circuit changes when the current flow in the circuit changes. Due to the change of flux, an emf is induced in the circuit. This emf is time rate of change of current. $V \alpha \frac{di}{dt} \quad OR \quad V = L\frac{di}{dt}$ The constant of proportionality L is known as the self-inductance. $L = N\phi / i$			
9	**Define mutual inductance.**	2	3	K2
	Mutual inductance is the property associated with two or more coils, which are in close proximity. The two coils are arranged in such a way that an emf is induced in the second coil due to the change in flux in the first coil and vice versa. $V \alpha \frac{di}{dt} \quad OR \quad V = M\frac{di}{dt}$ The constant of proportionality M is known as the Mutual inductance. $M = N_2\phi_{12} / i_1$			

10	**Define coefficient of coupling.**	2	3	K2
	Co-efficient of coupling is defined as the fraction of the magnetic flux produced by the current in one coil that links the other coil. The amount of coupling between two inductively coupled coils is expressed in terms of the coefficient of coupling, which is defined as $k = M/\sqrt{L_1 L_2}$ Where, M =Mutual inductance between the coils L_1 =Self-inductance of the first coil L_2 = Self-inductance of the second coil			
11	**Give the dot rules.**	2	3	K2
	The dot rule is as follows: (i) When the assumed currents both enter or both leave a pair of coupled coils by the dotted terminals, the signs on the L-terms, but (ii) If one current enters by a dotted terminal while the other leaves by a dotted terminal, the signs on the M-turns will be opposite to the signs on the L-terms.			
12	**What is tuned circuit?**	2	3	K2
	In a coupled circuit, when capacitors are added to primary and secondary of coupled coils to resonate the coils to achieve maximum power transfer condition then the coupled circuit is called tuned coupled circuit.			
13	**Define single tuned circuit and double tuned circuit**	2	3	K2
	In a coupled circuit when a capacitor is added to secondary coil to resonate the secondary, the coupled circuit is called single tuned coupled circuit. In a coupled circuit when capacitors are added both to primary and secondary coils to resonate the primary and secondary, the coupled circuit is called double tuned coupled circuit.			
14	If a coil of 800 μH is magnetically coupled to another coil of 200 μH . The coefficient of coupling between two coils is 0.05. Calculate the inductance if two coils are connected in series aiding, series opposing, parallel aiding and parallel opposing.	2	3	K3
	(ii) k=0.05 $M = K\sqrt{L_1 L_2}$ = 0.02mH In series aiding $L_{eq} = L_1 + L_2 + 2M$ In series opposing $L_{eq} = L_1 + L_2 - 2M$ **Answers:** Series aiding, L_{eq}= 1.04mH Series opposing, L_{eq}=0.96mH Parallel aiding $Leq = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$ Parallel opposing $Leq = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$ **Answers:** Parallel aiding, L_{eq}= 0.166mH Parallel opposing, L_{eq}=0.153mH			
15	A coil of inductance of value 300 mH is connected in series with another coil. The total inductance is 800 mH. When one coil is reversed, the total inductance is 400 mH. Find the coefficient of coupling between the coils.	2	3	K3
	In series aiding $L_{eq} = L_1 + L_2 + 2M$ In series opposing $L_{eq} = L_1 + L_2 - 2M$			

	L1=300mH Substitute in above eqn we get, L2+2M=500mH L2-2M=100 mH solving these two eqns we get **L2=300mH** By substituting L1,L2 values in any of the above eqn we get, M=100mH $M = K\sqrt{L_1 L_2}$ **K=0.3333**			
	PART – B (12 Mark Questions with Key)			
S.No	**Questions**	**Mark**	**COs**	**BTL**
1	**Two coupled coils with L_1=0.02H, L_2=0.01H and K=0.5 are connected in four different ways, series aiding, series opposing and parallel with both arrangements of the winding sense. What are the four equivalent inductances?**	12	3	K3
	$M = K\sqrt{L_1 L_2}$ = 7.07mH In series aiding $L_{eq} = L_1 + L_2 + 2M$ In series opposing $L_{eq} = L_1 + L_2 - 2M$ Parallel aiding $Leq = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$ Parallel opposing $Leq = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$ (i) Series aiding= 0.044 H (ii) Series opposing= 0.016 H (iii) Parallel aiding= 9.4mH (iv) Parallel opposing = 3.41mH	4 4×2 =8		
2	**In the given circuit find the phasor voltage V_2** 	12	3	K3

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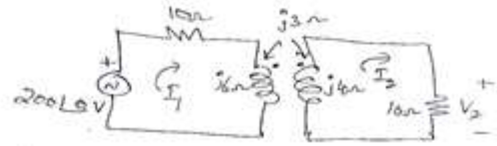
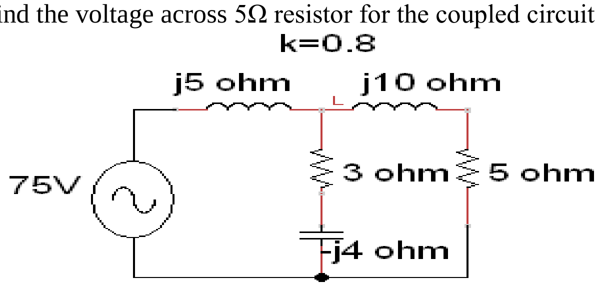
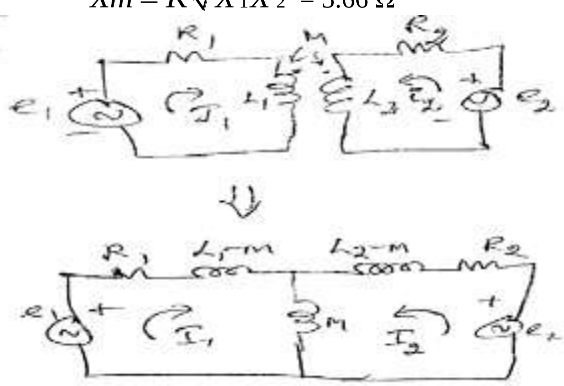


Soln: We can write mesh equations directly
$(10+j6) I_1 - j3 I_2 = 200\angle 0$ —— ①
$(10+j4) I_2 - j3 I_1 = 0$ —— ②
from ② ⇒ $(10+j4) I_2 = j3 I_1$
$I_1 = \frac{(10+j4) I_2}{j3}$
Substituting in eqn ①.
$\frac{(10+j4)}{j3}(10+j6) I_2 - j3 I_2 = 200\angle 0$
$\frac{(10+j4)(10+j6) I_2 - j^2 3^2 I_2}{j3} = 200\angle 0$
$((10+j4)(10+j6)+9) I_2 = 200\angle 0 \cdot j3$
$= 200\angle 0 \cdot 3\angle 90$
$(85+j100) I_2 = 600\angle 90$
$I_2 = \frac{600\angle 90}{85+j100}$
$= \frac{600\angle 90^\circ}{131.2\angle 49.6} = 4.57\angle 40.4 A$
`V_2= 10 I_2 = **45.7< 40.4 V**
</td><td>6</td><td></td><td></td></tr>
<tr><td>3</td><td>Find the voltage across 5Ω resistor for the coupled circuit given.


</td><td>12</td><td rowspan="2">3</td><td rowspan="2">K3</td></tr>
<tr><td></td><td>$M = K\sqrt{L_1 L_2}$ $Xm = K\sqrt{X_1 X_2}$ = 5.66 Ω


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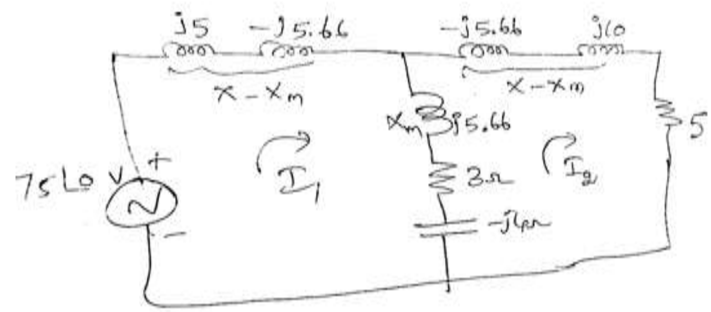


Method of Inspection (Mesh Method)

$$\begin{bmatrix} 3-j4+j\cancel{5.66}-j\cancel{5.66}+j5 & -(3-j4+j5.66) \\ -(3-j4+j5.66) & 5+j10-j\cancel{5.66}+j\cancel{5.66}+3-j4 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 75\angle 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3+j1 & -3-j1.66 \\ -3-j1.66 & 8+j6 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 75\angle 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3+j1 & -3-j1.66 \\ -3-j1.66 & 8+j6 \end{vmatrix} = 11.8+j16.04 = 19.9\angle 53.7$$

$$\Delta_2 = \begin{vmatrix} 3+j1 & 75\angle 0 \\ -3-j1.66 & 0 \end{vmatrix} = 257.3\angle 28.96$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{257.3\angle 28.96}{19.9\angle 53.7} = 12.93\angle -24.7 \text{ A}$$

$\therefore$ Voltage across $5\Omega = 5 I_2$

$$= 5 \times 12.93\angle -24.7$$

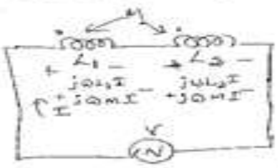
$$= 64.7\angle -24.7 \text{ V}$$

	(solution above)	3		
4	**Derive the equivalent inductance of the conductively connected mutually coupled circuits in (i) Series connection (ii) Parallel connection**	12	3	K3

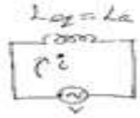
6

1. Series Connection < Series aiding or cumulative / Series opposition or Differential
2. parallel Connection < parallel aiding or Cumulative / parallel opposition or Differential.

① (a) Series aiding:



$L_{eq} = L_a$



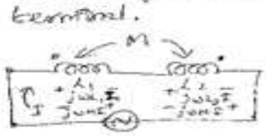
- Current is entering both the coils at the dotted terminals.
- Called series aiding combination.
- Mesh Equation (KVL)

$$V = j\omega L_1 I + j\omega M I + j\omega L_2 I + j\omega M I$$
$$= j\omega I(L_1 + M + L_2 + M)$$
$$= j\omega I(L_1 + L_2 + 2M)$$
$$\frac{V}{I} = j\omega(L_1 + L_2 + 2M)$$
$$= Z$$

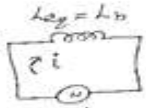
Leq = L_1+L_2+2M

1(b) Series opposition (bucking)

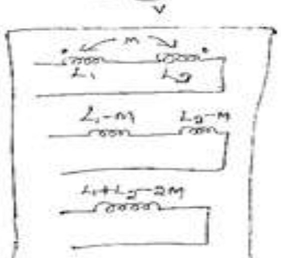
- current is entering first coil at dotted terminal & leaving the other coil at dotted terminal.



$L_{eq} = L_b$



$$V = j\omega L_1 I - j\omega M I + j\omega L_2 I - j\omega M I$$
$$= j\omega I(L_1 + L_2 - 2M)$$
$$\frac{V}{I} = j\omega(L_1 + L_2 - 2M)$$
$$= Z$$
$$L_{eq} = L_1 + L_2 - 2M$$



Equivalent Inductance

$$\boxed{L_b = L_1 + L_2 - 2M}$$

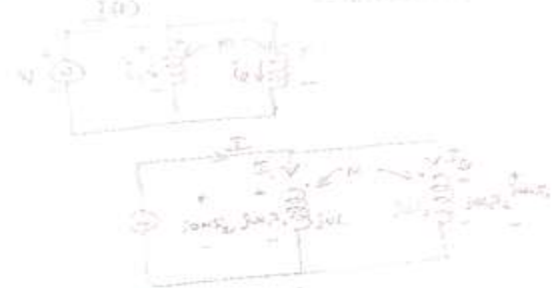
NOTE:

[Equivalent inductance in the series aiding combination is more than that in series opposing combination by an amount = 4M]

2(a) Parallel aiding:

– both currents I_1 & I_2 enter [illegible]
at dotted terminals.

Assume [illegible]



Mesh Equations

$V = j\omega L_1 I_1 + j\omega M I_2$ —— ①

$V = j\omega L_2 I_2 + j\omega M I_1$ —— ②

In matrix form

$$\begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V \\ V \end{bmatrix}$$

$$\therefore \Delta = \begin{vmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{vmatrix} = j^2\omega^2 L_1 L_2 - j^2\omega^2 M^2$$

$$= j^2\omega^2 (L_1 L_2 - M^2)$$

$$= -\omega^2 (L_1 L_2 - M^2)$$

$$= \underline{\omega^2 (M^2 - L_1 L_2)}$$

$$\Delta_1 = \begin{vmatrix} V & j\omega M \\ V & j\omega L_2 \end{vmatrix} = V j\omega L_2 - V j\omega M$$

$$= \underline{V j\omega (L_2 - M)}$$

$$\Delta_2 = \begin{vmatrix} j\omega L_1 & V \\ j\omega M & V \end{vmatrix} = V j\omega L_1 - V j\omega M$$

$$= \underline{V j\omega (L_1 - M)}$$

$$\therefore I_1 = \frac{\Delta_1}{\Delta} = \frac{V j\omega (L_2 - M)}{\omega^2 (M^2 - L_1 L_2)}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{V j\omega (L_1 - M)}{\omega^2 (M^2 - L_1 L_2)}$$

Total current

$$I = I_1 + I_2$$

$$= \frac{V j\omega (L_2 - M)}{\omega^2 (M^2 - L_1 L_2)} + \frac{V j\omega (L_1 - M)}{\omega^2 (M^2 - L_1 L_2)}$$

$$= \frac{V j\omega}{\omega^2} \left(\frac{L_2 - M + L_1 - M}{M^2 - L_1 L_2} \right)$$

$$= \frac{Vj\omega}{\omega^2}\,\frac{(L_1+L_2-2M)}{(M^2-L_1L_2)}$$

$$I = \frac{Vj\omega\,(L_1+L_2-2M)}{\omega^2\,(M^2-L_1L_2)}$$

Total input Impedance $Z = \frac{V}{I}$

$$Z = \frac{\cancel{V}\omega^{\cancel{2}}(M^2-L_1L_2)}{\cancel{V}j\cancel{\omega}(L_1+L_2-2M)}$$

$$= \frac{-j\omega(M^2-L_1L_2)}{L_1+L_2-2M}$$

$$Z = \frac{j\omega(L_1L_2-M^2)}{(L_1+L_2-2M)} \quad \text{--- (3)}$$

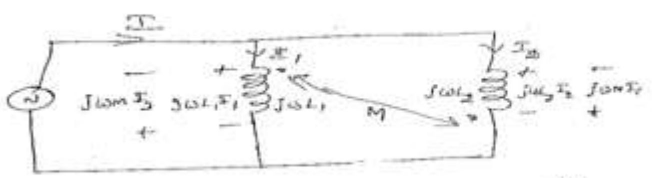
But, L_q be the equivalent of the combination of inductance.

Then, $Z = jX_L = jL_{eq}\omega$
$= jL_q\omega$ --- (4)

From (3) & (4)

Equivalent Inductance $\boxed{L_q = \frac{L_1L_2-M^2}{L_1+L_2-2M}}$

2 (b) Parallel opposition:



$$V = j\omega L_1 I_1 - j\omega M I_2 \quad \text{--- (1)}$$
$$V = j\omega L_2 I_2 - j\omega M I_1 \quad \text{--- (2)}$$

In matrix form

$$\begin{bmatrix} j\omega L_1 & -j\omega M \\ -j\omega M & j\omega L_2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V \\ V \end{bmatrix}$$

$$\Delta = \begin{vmatrix} j\omega L_1 & -j\omega M \\ -j\omega M & j\omega L_2 \end{vmatrix} = j^2\omega^2 L_1L_2 - j^2\omega^2 M^2 = -\omega^2 L_1L_2 + \omega^2 M^2 = \omega^2(M^2-L_1L_2)$$

$$\Delta_1 = \begin{vmatrix} V & -j\omega M \\ V & j\omega L_2 \end{vmatrix} = j\omega L_2 V + j\omega M V = j\omega V(L_2+M)$$

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$$\Delta_2 = \begin{vmatrix} j\omega L_1 & V \\ -j\omega M & V \end{vmatrix} = j\omega L_1 V + j\omega M V = j\omega V(L_1+M)$$
$$\therefore I_1 = \frac{\Delta_1}{\Delta} = \frac{j\omega V(L_2+M)}{\omega^2(M^2-L_1L_2)}$$
$$I_2 = \frac{\Delta_2}{\Delta} = \frac{j\omega V(L_1+M)}{\omega^2(M^2-L_1L_2)}$$
Total current
$$I = I_1+I_2$$
$$= \frac{j\omega V}{\omega^2}\left(\frac{L_2+M+L_1+M}{M^2-L_1L_2}\right)$$
$$= \frac{j\omega V}{\omega^2}\frac{(L_1+L_2+2M)}{(M^2-L_1L_2)}$$
$$Z = \frac{V}{I} = \frac{\cancel{V}\cdot\omega^{\cancel{2}}(M^2-L_1L_2)}{j\cancel{\omega}\cancel{V}(L_1+L_2+2M)}$$
$$= \frac{-j\omega(M^2-L_1L_2)}{L_1+L_2+2M}$$
$$Z = \frac{j\omega(L_1L_2-M^2)}{L_1+L_2+2M} \quad \text{—— (3)}$$
But w.k.t
$$Z = j\omega L_{eq} = j\omega L_b \quad \text{—— (4)}$$
Comparing (3) and (4)
$$Leq = \frac{L_1L_2-M^2}{L_1+L_2+2M}$$
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<tr><td>5</td><td>In a series fed double tuned transformer circuit both primary and secondary are tuned to the same resonant frequency of 10^5 rad/sec. The maximum output voltage across the capacitor is 24V and is obtained by varying K. If the inductance and the resistance of the primary circuit are 4μH & 0.1Ω respectively and secondary circuit are 50μH & 1Ω respectively. Calculate the supply voltage.</td><td>12</td><td rowspan="2">3</td><td rowspan="2">K3</td></tr>
<tr><td></td><td>
Soln $\omega_r = \frac{1}{\sqrt{L_1C_1}} = \frac{1}{\sqrt{L_2C_2}} = 10^5$ rad/sec

$E_o = 24V$; $L_1 = 4\mu H$ $R_1 = 0.1\Omega$

$E_g = ?$ $L_2 = 50\mu H$ $R_2 = 1\Omega$

R_g = not given So '0' value is assumed.

Maximum, $E_{o,c} = \frac{E_g}{2\omega_r C_2\sqrt{R_1R_2}}$ —— (1)

$$\frac{1}{\sqrt{L_2C_2}} = 10^5 \text{ rad/sec}$$
$$\Rightarrow \sqrt{C_2} = \frac{1}{\sqrt{L_2}\times10^5}$$
$$C_2 = \frac{1}{L_2\times10^{10}} = \frac{1}{50\times10^{-6}\times10^{10}}$$
</td><td>6</td></tr>
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	$= \frac{1}{50\times10^{-6}\times10^{10}} = \frac{1}{50\times10^{4}}$ $= 2\times10^{-6}$ F $= \underline{\underline{2\ \mu F}}$ Eqn ①, $\Rightarrow 24 = \frac{E_g}{2\times10^{5}\times2\times10^{-6}\sqrt{0.1\times1}}$ $E_g = \underline{\underline{3.036}}\ V = 3.04\ V$ (approx)			
6	**A double tuned circuit is tuned to a frequency of 750 Hz. When excited by a voltage source at critical coefficient of coupling the maximum output voltage across C_2 is 20V. Determine the coefficient of coupling and the source voltage. The circuit parameters are Q_1=6; R_1=10Ω; Q_2=10; R_1=90Ω**	12	3	K3
	Solun Given $E_{o,c} = 20V$ $K_c = ?$ $E_g = ?$ $\omega_r = 750$ $R_1 = 10$, $R_2 = 90$ $Q_1 = 6$, $Q_2 = 10$ $K_c = \frac{1}{\sqrt{Q_1 Q_2}} = \frac{1}{\sqrt{6\times10}} = \underline{\underline{0.1291}}$ (Ans) Voltage at Critical coupling. $E_{o,c} = \frac{E_g}{2\omega_r C_2\sqrt{R_1R_2}}$ $E_g = E_{o,c}\times2\omega_r C_2\sqrt{R_1R_2}$ $= 20\times2\times750\times C_2\sqrt{10\times90}$ $= ?$ $C_2 = ?$	6		

$$\boxed{Q_2 = \frac{\omega_r L_2}{R_2}}$$

$$L_2 = \frac{Q_2 R_2}{\omega_r} = \frac{10 \times 90}{750} = 1.2H$$

At resonance,

$$\omega_r L_2 = \frac{1}{\omega_r C_2} \qquad \omega_r = \frac{1}{\sqrt{L_2 C_2}}$$

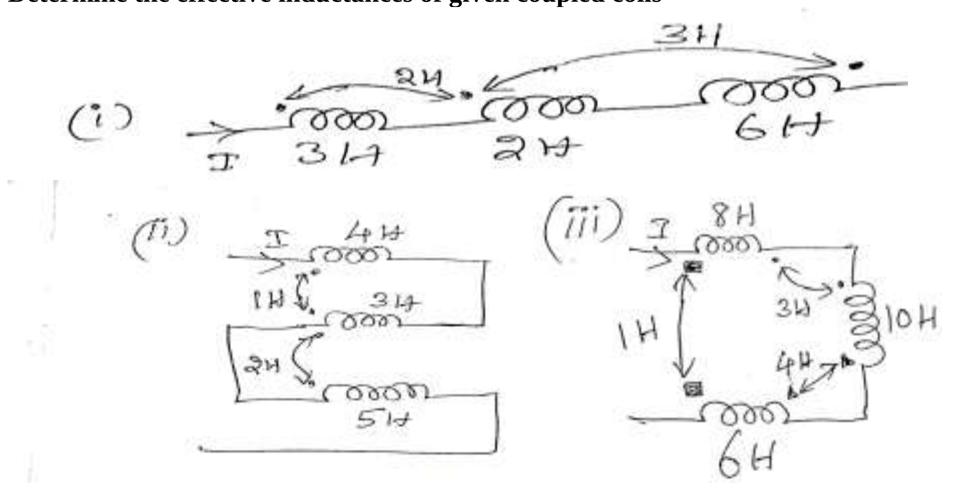
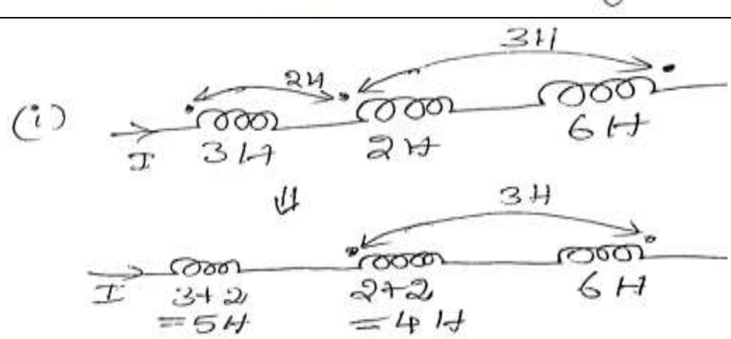
$$C_2 = \frac{1}{\omega_r^2 L_2} = \frac{1}{750^2 \times 1.2}$$

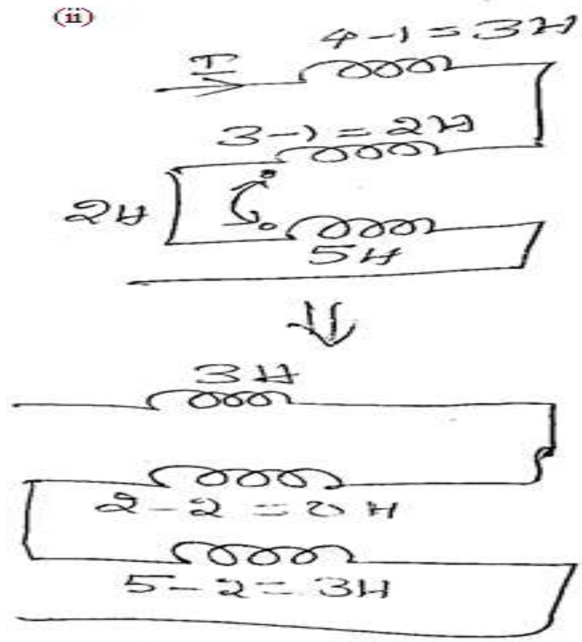
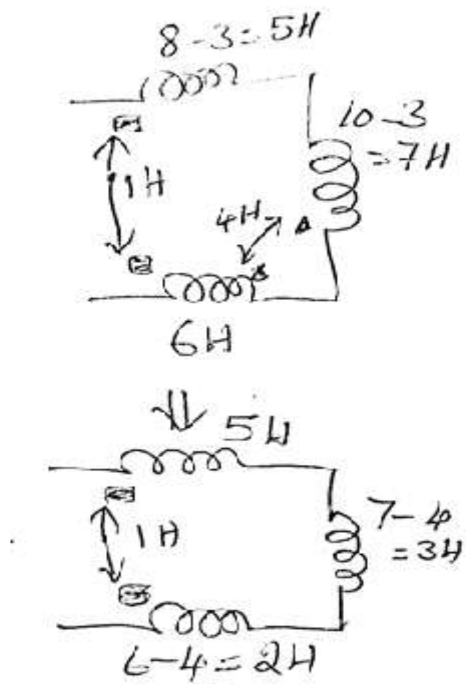
$$= \underline{1.4815 \ \mu F}$$

$$E_g = 20 \times 2 \times 750 \times 1.4815 \times 10^{-6} \times \sqrt{10 \times 90}$$

$$= \underline{1.3335} \ V \quad \text{(Ans)}$$

PART – C (20 Mark Questions with Key)

S.No	Questions	Mark	COs	BTL
1	**Determine the effective inductances of given coupled coils** 	20	3	K3
		6+7+7 =20		

	Leq = 5 + (4-3) + (6-3) = 5+1+3 = **9 H**  Leq = 3+ 0 +3 = **6 H**  Leq = (5-1) + 3 + (2-1) = 4+3+1 = **8 H**			
2	**In a series fed double tuned circuit a maximum voltage gain of 20 was obtained at a resonance frequency of 10^6 rad/sec. The capacitance in the primary circuit is 2μF. The maximum output voltage at resonance is 50V. Calculate (i) supply voltage (ii) primary and secondary self-inductances (iii) the critical coefficient of coupling (iv) the capacitance in the secondary circuit assume that the primary and secondary resistances are 1Ω and 4Ω respectively.**	20	3	K3

	soln Max. voltage gain $= A_m = 20$ Resonant freq $= \omega_r = 10^6$ rad/sec $C_1 = 2\,\mu F$ $E_{oc} = 50V$ $R_g = 0$; $R_1 = 1\Omega$; $R_2 = 4\Omega$ (i) $E_g = ?$ (ii) L_1 & $L_2 = ?$ (iii) $K_c = ?$ (iv) $C_2 = ?$ w.k.t $A_m = \frac{E_o}{E_g} = 20$ $E_g = \frac{E_o}{20} = \frac{50}{20} = 2.5V$ $\boxed{E_g = 2.5V}$ 1° at resonance $\omega_r = \frac{1}{\sqrt{L_1 C_1}}$ 2° at resonance $\omega_r = \frac{1}{\sqrt{L_2 C_2}}$ $10^6 = \frac{1}{\sqrt{2\times10^{-6}\times L_1}}$ $10^{12} = \frac{1}{2\times10^{-6} L_1} \Rightarrow L_1 = \frac{1}{2\times10^{-6}\times10^{12}}$ $= 0.5\times10^{-6} H$ $= 0.5\,\mu H$ $\boxed{L_1 = 0.5\,\mu H}$ $E_{oc} = \frac{E_g}{2\omega_r C_2\sqrt{R_1R_2}}$ $50 = \frac{2.5}{2\times10^6\times C_2\sqrt{1\times4}}$ $C_2 = \frac{2.5}{50\times2\times10^6\times2}$ $= 1.25\times10^{-8} F$ $\boxed{C_2 = 1.25\times10^{-8} F}$ or $12.5\,nF$ $\omega_r = \frac{1}{\sqrt{L_2C_2}}$ $\omega_r^2 = \frac{1}{L_2C_2} \Rightarrow L_2 = \frac{1}{\omega_r^2 C_2}$ $L_2 = \frac{1}{(10^6)^2\times1.25\times10^{-8}}$ $= 80\times10^{-6} H$ $\boxed{L_2 = 80\,\mu H}$ $M_c = K_c\sqrt{L_1L_2}$ $M_c = \frac{\sqrt{R_1R_2}}{\omega_r} = \frac{\sqrt{1\times4}}{10^6}$ $= 2\times10^{-6} H$ $\boxed{M_c = 2\,\mu H}$ $\therefore K_c = M_c/\sqrt{L_1L_2}$ $= \frac{2\times10^{-6}}{\sqrt{0.5\times10^{-6}\times80\times10^{-6}}} = 0.3162$ (Ans)	10		